Extended Reference: RC, RL, and RLC Circuits

🔧 1. Impedance Basics (AC / Phasor Domain)

- Resistor: $Z_R = R$
- Capacitor: $Z_C = \frac{1}{j\omega C} = -j\frac{1}{\omega C}$
- Inductor: $Z_L = j\omega L$
- Series: $Z_{\text{total}} = Z_1 + Z_2 + \ldots$
- Parallel: $\frac{1}{Z_{\text{total}}} = \frac{1}{Z_1} + \frac{1}{Z_2} + \ldots$

Where $j = \sqrt{-1}$ and $\omega = 2\pi f$

⚡ 2. RC Circuits

(a) RC Series

- **Impedance:** $Z = R + \frac{1}{j\omega C}$
- **Voltage divider (AC):**
  - $V_C = V_{in} \cdot \frac{1/j\omega C}{R+1/j\omega C}$
  - $V_R = V_{in} \cdot \frac{R}{R+1/j\omega C}$
- **Low-pass filter cutoff frequency:**
  $f_c = \frac{1}{2\pi RC}$

(b) RC Parallel

- **Impedance:** $Z = \left(\frac{1}{R} + j\omega C\right)^{-1}$
- **High-pass filter behavior** (output across resistor)

⚙️ 3. RL Circuits

(a) RL Series

- **Impedance:** $Z = R + j\omega L$
- **High-pass filter behavior** (voltage across inductor)

(b) RL Parallel

- **Impedance:** $Z = \left(\frac{1}{R} + \frac{1}{j\omega L}\right)^{-1}$
- **Low-pass filter behavior** (output across inductor)

🔁 4. RLC Series Summary

- **Impedance:**
  $Z = R + j\left(\omega L - \frac{1}{\omega C}\right)$
- **Resonant frequency:**
  $\omega_0 = \frac{1}{\sqrt{LC}}$

🧠 5. Use Cases Summary

Circuit Type	Frequency Role	Typical Use Case
RC Series	Low-pass	Anti-aliasing, envelope detection
RC Parallel	High-pass	AC coupling, RF deblocking
RL Series	High-pass	RF filtering, snubbers
RL Parallel	Low-pass	Motor drives, soft start circuits
RLC Resonant	Band-pass/notch	Tuned filters, antennas

🔍 6. Definitions

Time Constants

- $\tau_{RC} = RC$
- $\tau_{RL} = \frac{L}{R}$

Quality Factor

- **RC Circuit:** $Q = \omega RC$
- **RL Circuit:** $Q = \frac{\omega L}{R}$
- **RLC Circuit:** $Q = \frac{\omega_0 L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$

Bandwidth (RLC)

- $\Delta f = \frac{f_0}{Q}$

🔄 7. Evolution of j and -1 Around the Unit Circle

Rotation	Angle (rad)	j^n	Equivalent
0	0	1	Real +ve
1	$\frac{\pi}{2}$	j	Imag +ve
2	π	-1	Real -ve
3	$\frac{3\pi}{2}$	$-j$	Imag -ve
4	2π	1	(cycle repeats)

🧠 Euler's identity:
$e^{j\theta} = \cos(\theta) + j\sin(\theta)$

➕➖ 8. Trig Function Signs by Quadrant

Quadrant	Angle Range	sin	cos	tan	arctan
I	0° to 90°	$+$	$+$	$+$	$+$
II	90° to 180°	$+$	$-$	$-$	$+$ (angle still positive)
III	180° to 270°	$-$	$-$	$+$	$-$
IV	270° to 360°	$-$	$+$	$-$	$-$

🧠 Memory aid: **"All Students Take Calculus"**

📶 9. Impedance Matching

- **Goal:** Maximize power transfer, minimize reflection or loss.

- **Maximum power transfer condition:**
  $Z_{\text{load}} = Z_{\text{source}}^*$
- **For purely resistive systems:**
  $R_L = R_S$

Matching Methods:

Series/Parallel L and C

- Use **series or parallel** capacitors or inductors to cancel reactance:
  - **To cancel inductive reactance:** add a series capacitor.
  - **To cancel capacitive reactance:** add a series inductor.
- Match resistive parts via resistors or transformers.

L-Networks (2-element matchers)

- Match any real impedance to another using **L-section**:
  - One capacitor and one inductor.
  - One is **series**, one is **shunt**.
  - Choice depends on whether $Z_L > Z_S$ or vice versa.

Transformers

- Use turns ratio:

$$Z_L' = Z_L \cdot \left(\frac{N_P}{N_S}\right)^2$$

Quarter-Wave Transformers (RF)

- Transmission line length $\lambda/4$ with characteristic impedance:

$$Z = \sqrt{Z_S Z_L}$$

Smith Chart (RF/analog matching visualization)

- Visual tool for matching complex impedances.

🔌 10. Power Factor

- **Power factor:**

$$\text{PF} = \cos(\theta) = \frac{P_{\text{real}}}{P_{\text{apparent}}}$$

- θ = phase angle between voltage and current.
- **PF = 1:** purely resistive (ideal).
- **PF < 1:** presence of reactive power → wasted energy.
- Lagging PF → inductive load.
- Leading PF → capacitive load.
- **Correction:** add capacitors to cancel inductive lag (and vice versa).

⚡ 11. AC Power Types

True Power (Real Power)

- $P = V_{\text{rms}} I_{\text{rms}} \cos(\theta)$
- Units: **Watts (W)**
- Power that actually does work (heating, mechanical output, etc.)

Reactive Power

- $Q = V_{\text{rms}} I_{\text{rms}} \sin(\theta)$
- Units: **Volt-Amps Reactive (VAR)**
- Power that oscillates back and forth between source and reactive elements (L/C).

Apparent Power

- $S = V_{\text{rms}} I_{\text{rms}}$
- Units: **Volt-Amps (VA)**
- Vector sum of real and reactive power:

$$S = P + jQ$$

- Magnitude:

$$|S| = \sqrt{P^2 + Q^2}$$

📦 12. Real Device Examples and Applications

Device Type	Role of RLC / Matching / Power
SMPS (Switching PS)	LC filters, impedance control
Audio Amplifier	RL output filtering, quality factor (Q), power factor
Induction Motor	Low power factor, requires capacitive correction
Grid Inverter	Measures P, Q, and S via analog/dedicated IC
RF Antenna	Requires precise impedance matching (50 Ω typical)
Wireless Charger	Resonant LC matching (RLC tank circuit)
Power Meter (ICs)	Calculates power factor, P, Q, S using analog circuitry

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